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Trench effects on lateral p-y relations for pipelines embedded in stiff soils and rocks

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Abstract

Existing relationships for lateral backfill pressures on pipelines assume that the trench is adequately wide to contain the failure surface. This condition is commonly violated in design and construction practice, putting at risk the pipeline safety. In this context, size and shape effects for trenches excavated in stiff soils and rocks, are numerically investigated, through experimentally-calibrated parametric analyses. It is shown that, for narrow trenches, ultimate pressures and yield displacements may increase up to an order of magnitude compared to “infinite-trench” values, while excavation of inclined walls reduces the above detrimental effects. Simplified relations are developed to aid pipeline design.

1. Introduction

It is widely acknowledged that trenches backfilled with loose to medium dense sand can drastically reduce design demands for buried pipelines subjected to permanent ground movement (e.g. fault rupture) in the core of stiff soil and rocky terrain. The reason is that the magnitude of soil pressures imposed to the pipeline is controlled by the properties of the backfill material and not by those of the much stiffer (in most cases) natural surrounding ground. Evidently, for the previous statement to be valid, the trench should be adequately large in order to fully contain the mobilized failure surface.

It is noteworthy that the potential effects of trench size are acknowledged in current design guidelines [1–5], but only in a qualitative way. For instance, according to ALA-ASCE (2001), the backfill soil properties for the evaluation of soil pressures can only be used if the size of

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the trench is "*adequate*". Even though, and also despite the large number of studies dedicated to the response of buried pipelines [e.g. 6–14], research related to trench size effects remained limited until recently.

To fill this gap, Kouretzis et al. [15] and Chaloulos et al. [16] investigated systematically the extent of sand backfill failure for the case of laterally displaced pipelines (e.g. at strike-slip fault crossings), with the aid of experimentally calibrated linear elastic-perfectly plastic numerical analyses. It was thus shown that, common trench sections rarely ensure the unobstructed development of the failure surface inside the backfill sand. For instance, in the common case of a $D=30\text{in}$ (0.76m) diameter pipeline embedded at $H=1.50\text{m}$ average depth (i.e. $H/D\approx 2.0$), the required net half-width of the trench for free development of the failure surface within the sand backfill exceeds 3m , not including the fault-induced displacement, as compared to the $0.2\text{--}0.5\text{m}$ allowed in common practice. Under these conditions, the pipeline response is not controlled by the backfill, but by the much stiffer surrounding soil thus increasing both the pressures applied to the pipeline and the associated pipeline strains.

In light of the above, the present paper focuses upon the analytical computation of increased ultimate soil pressures and lateral yield displacements in the case of "narrow" and "shallow" trenches, i.e. when trench dimensions (width, depth and side wall inclination) are not adequate for un-hindered failure within the sand backfill. For this purpose, the numerical methodology that has been developed and verified in Chaloulos et al. [16] is now applied parametrically, for various backfill soil properties, pipeline diameters and embedment depths, in order to evaluate and incorporate trench boundary effects in the design of pipelines with the commonly applied "*beam on Winkler soil springs*" method. More specifically, the final output of the investigation is a set of equations that modify the Winkler soil spring characteristics according to the trench size and shape. To increase the application range of the proposed equations, trench size and shape effects are expressed in the form of

correction factors which can be readily combined with existing relations for pipelines embedded in infinitely extending sand layers. To aid independent reading of the paper, Section 2 repeats briefly the numerical methodology that was used to simulate the problem, as well as the failure mechanism for laterally displaced pipes in uniform sand, obtained by Chaloulos et al. [16].

2. Outline of Numerical Analyses and Results

2.1 Numerical Methodology

Figure 1 shows the finite difference mesh and the backfill sand that was used for the bulk of the parametric analyses: a cylindrical pipe section with diameter D is embedded at depth H , measured from the center of the pipeline, in an artificial trench backfilled with sand. The pipeline is displaced laterally to a maximum displacement $y=y_{\max}$. The effect of trench geometry on the development of soil pressures was investigated by varying distances x and d , also shown in Figure 1. More specifically, the horizontal distance x defines the semi-width of the trench in the direction of pipeline movement (left side in fig. 1), and it is measured from the center of the pipeline (at its displaced position) to the lateral boundary. For the reference "infinite-trench" conditions, x was equal to $16D$ and was then step-by-step reduced to $0.75D$ for the most narrow trench conditions. Note that in the direction opposite to the pipeline movement (right side in fig. 1), the semi-width of the trench was kept constant and equal to $6.5D$ throughout the parametric investigation, based on the results of sensitivity analyses which showed that the exact location of the wall at this side of the trench, even for the small lateral distances used in practice, does not affect the development of soil pressures. The vertical distance d is measured from the bottom of the pipeline to the base of the trench. Finally, in the majority of the parametric analyses, the trench has vertical side walls, while a number of parametric analyses is also performed for outwards inclined trench walls. In this

trapezoidal trench geometry, the aforementioned horizontal dimensions refer to the horizontal plane passing through the pipeline axis.

All parametric analyses were performed with the finite difference code FLAC v7.0 [17]. The large strain formulation mode was activated, in combination with a mesh rezoning technique [16], in order to account for the large lateral displacement of the pipe section. Following sensitivity analyses the mesh was discretized into square elements of size $0.1D$. Special attention was placed regarding the selection of proper boundary constraints. Namely, for the simulation of the experiments, the selection was based on comparative analyses either with rollers or with hinges, which revealed that the latter provided a more consistent agreement between experimental results and numerical predictions. For the parametric study, the selection of proper boundary constraints was guided by the nature of the problem. Namely, as the analysis assumes that the natural soil is much stiffer than the backfill sand, it is reasonable to expect that the sides of the excavated trench will be rough and failure will take place within the backfill and not along the backfill-trench interface. Consequently, the side boundaries were considered as “rough” and simulated with hinges. It is also of interest to note that, parallel analyses performed for “smooth” side boundaries, i.e. with vertical rollers instead of hinges, have shown that application of hinges is more damaging for the pipeline (leads to larger spring reactions), suggesting that the selection of hinged boundary constraints is conservative.

The backfill material was given the characteristics of Cornell filter sand, i.e. the material of the model experiments that were used in order to calibrate the numerical methodology, with unit weight $\gamma=14.8$ and 16.4kN/m^3 for loose and medium density respectively [18]. Note that the use of dense sand backfill is not recommended by design codes, as it would unnecessarily increase soil pressures on the pipeline, and consequently it was not considered in this study. The analyses for both sand backfill densities were performed using

the elastic-perfectly plastic Mohr-Coulomb model. The friction angle was computed from the critical state value $\phi_{cr}=31^\circ$ that was obtained from direct shear tests on Cornell Sand [18] and was consequently modified to $\phi_{cr,PS}=37^\circ$ in order to account for the actually prevailing plane strain conditions [19]. The critical state angle of dilation was $\psi=0$. The choice of this approach was straightforward for the case of loose sand backfill. Furthermore, for the case of medium dense sand backfill, it was based on the observation that the lateral pipeline translation causes relatively large deformations in the backfill (re-meshing was found necessary in all analyses) and consequently full mobilization of the failure surface is much closer to critical state rather than to peak strength conditions. Note that the same assumption has been also adopted by Kouretzis et al. [15] and Chaloulos et al. [16] for the numerical simulation of lateral pipeline translation into infinitely extending loose and medium dense sand backfill. Finally, the Young modulus varied with vertical effective stress as [7]:

$$E = 2 \cdot 10^{-13.97} \cdot \left(\gamma \cdot \sigma_v^{0.0378} \right)^{13.7} \quad (1)$$

where γ the unit weight [kN/m³] and σ'_v the vertical effective stress [kPa]. Note that eq. (1) is readily reduced to a more recognizable Hardin type formula [i.e. $E=A \cdot f(e) \cdot (\sigma'_v)^{0.5}$] if the unit weight is written as $\gamma=\gamma_s/(1+e)$, where γ_s is the unit weight of the solid soil particles ($\gamma_s \approx 26\text{--}27\text{kN/m}^3$) and e is the void ratio. Moreover, this empirical equation has been developed by O'Rourke [7], strictly in connection with elastic-perfectly plastic analyses of embedded pipelines using the Mohr-Coulomb failure criterion for the sand backfill, and was thus preferred over other (more general) similar relations in order to optimize the accuracy of the numerical computations. Following the current practice for soil spring characterization (e.g. [1-5]), the pipe was conservatively simulated as a rigid body, i.e. ignoring any possible pipe ovalization effects. Furthermore, it was assumed that it is fixed at the vertical direction. Note that sensitivity analyses were also performed without vertical constraints on the pipe but yielded negligible differences. Interface elements were placed between the pipe and the soil

in order to simulate the relative pipe-soil slippage. Zero cohesion was assigned to that interface while the friction angle was set equal to one half that of the soil, i.e. $\varphi_{\text{int}}=1/2\varphi_{\text{cr,PS}}=18.5^\circ$, taking into account that interlocking between the pipeline steel and the sand backfill is minimal. It is in addition noted that, sensitivity analyses performed as part of the present as well as previous studies (e.g. [12]) have indicated that the predicted p-y response is not particularly sensitive to the assumed interface friction angle value.

The numerical methodology described above was calibrated against the experiments reported by Trautmann and O'Rourke [18], which involved lateral displacement of a $D=0.10\text{m}$ diameter straight pipeline inside a trench backfilled with Cornell sand. The geometry of the numerical model that was used for the verification of the numerical methodology was identical to that of the experiment, as described in detail in Yimsiri et al [12]. Namely, the length of the model was equal to 2.3m and the height was variable based on the H/D ratio, leaving approximately 300mm of space below the base of the pipeline. The pipeline was placed at a distance of 600mm from the right boundary and was pushed towards the left boundary.

A typical comparison between predictions and test results is shown in **Figure 2**, for shallow and deep embedment ratios, $H/D=1.5$ & 5.5 , as well as for loose and medium dense backfill sand. The agreement is fairly good, especially for the ultimate soil pressure which is of greater interest for the pipeline response assessment.

2.2 Shape and size of failure surface

As stated in the introduction, the shape and size of the failure surface was extensively investigated in the study by Chaloulos et al. [16]. Hence, the present section summarizes the main findings of that study, as they are considered necessary for the interpretation and understanding of the numerical results regarding trench effects. In short, the following three distinct failure modes were observed:

- General Shear failure (Type I): A wedge type, general shear failure surface develops which emerges to the ground surface (**Figure 3a**). This failure mode applies to shallow embedment ratios, below approximately $H/D=6.0$ and 4.8 for loose and medium dense backfill sand respectively.
- Local Shear failure (Type III): The failure mechanism has a circular shape and develops locally, around the pipe (**Figure 3c**). This mode of failure applies to large embedment ratios, beyond approximately $H/D=10$ and 9.5 for loose and medium dense sands respectively.
- Intermediate shear failure (Type II): For intermediate embedment depths, the failure mechanism is not well defined, as it progressively switches from general (*Type I*) to local (*Type III*) shear failure (**Figure 3b**).

In all analyses, the magnitude of the applied displacement was equal to $0.75D$, a value that allowed for the load-displacement curves to reach a plateau and the failure surface to be completely formed. The maximum width, x_{\max} , of the failure surface for each failure mode, measured from the displaced pipe axis (**Figure 3**), can be computed analytically as:

$$\text{Type I: } x_{\max} / D = 3 + 0.1 \cdot (H / D)^{c_1} \quad (2)$$

$$\text{Type II: } x_{\max} / D = 13.1 - 1.2 \cdot (H / D) \quad (3)$$

$$\text{Type III: } x_{\max} / D = c_2 \quad (4)$$

where $c_1=1.9$ & 2.40 and $c_2=1.10$ & 1.70 for loose and medium dense sand respectively. In a simpler way, although at some cost in accuracy, x_{\max} can be normalized against the embedment depth H and expressed as:

$$x_{\max} / H = 3.5 \cdot e^{-0.27 \cdot (H/D)} \quad (5)$$

The corresponding maximum depth, d_{\max} , and the inclination, θ_{\max} , of the failure surface can be also evaluated analytically, as:

$$d_{\max} / D = 0.20 - 0.30 \quad (6)$$

$$\theta_{\max} = 45 - 65^\circ \quad (7)$$

2.3 Load-Displacement (p-y) relations for lateral pipeline displacement

The gray continuous line in **Figure 4a** shows a typical load-displacement (p-y) curve obtained from the numerical analyses, where p (kN/m) denotes the soil pressure over a unit length of the pipeline. It may be observed that the shape of this curve resembles closely a hyperbola, and can be analytically expressed as:

$$y = \frac{p}{K_{\text{ini}}} \cdot \frac{1}{1 - p/p_{\text{ult}}} \quad (8)$$

where K_{ini} is the initial stiffness of the curve and p_{ult} is the ultimate soil pressure. In a (y/p) vs. y coordinate system, the above equation transforms into a straight line with slope $1/p_{\text{ult}}$ and y-intercept equal to $1/K_{\text{ini}}$. Thus, the two hyperbola parameters in Eq. 8 can be estimated directly by fitting a straight line to the numerical predictions in a (y/p) vs. y system of coordinates, as shown in **Figure 4b**.

When a bi-linear elastic-perfectly plastic p-y relation is used to model the soil spring response, as recommended in a number of pipeline design guidelines (e.g. ALA-ASCE 2001; PRCI 2009) and shown with dashed line in Figure 4a, the definition of p_{ult} should be accompanied by estimation of the corresponding ultimate (yield) displacement, i.e. the displacement at the transition point from the linear elastic to the perfectly plastic p-y response (y_{ult} in Figure 4a). To take into account the increased significance for the pipeline design of the post yield segment of the p-y response, the simplified bi-linear p-y relation was fitted to the aforementioned more exact hyperbolic relation at relatively high load levels,

namely at $p=0.70p_{ult}$ [e.g. 18, 20 & 21]. In that case, the yield displacement can be analytically computed from Eq 8 as $y_{ult} \approx 2.33 p_{ult}/K_{ini}$.

3. Effect of trench width

The effect of trench width (distance “x” in **Figure 1**) was first analyzed for trenches with vertical side walls which represent the standard construction practice today. A total of 160 parametric analyses were performed for both loose and medium backfill sand, as well as for a large range of embedment depth ratios between $H/D=1.5$ and 16. For each sand density and H/D value, the response was initially obtained for “infinite-trench” conditions i.e. where the side walls of the trench were placed at a very large distance from the pipeline axis ($x/D \approx 16$). In the sequel, the width of the trench was gradually reduced to a minimum normalized distance $x/D=0.75$ and ultimate soil pressures and displacements were compared to the reference “infinite-trench” values. In the majority of the analyses the pipe diameter was $D=0.10\text{m}$, while potential scale effects were evaluated by selectively repeating a number of the analyses for $D=0.70\text{m}$ with all other mesh dimensions proportionally scaled.

3.1 Increase of ultimate soil pressure, p_{ult}

The effect of trench width on ultimate soil pressures is illustrated in **Figure 5a** to **Figure 5c** for embedment depth ratios $H/D=1.5$, 6.5 and 11.5, corresponding to Type I, II and III failure modes respectively. Different symbols are used for loose and medium dense sand, as well as for pipe diameter $D=0.10$ and 0.70m . The vertical axis shows the ultimate soil pressure, p_{ult} , normalized against the reference “infinite-trench” value, $p_{ult,inf}$, while the horizontal axis shows the width of the trench x normalized against the maximum width of the failure surface x_{max} .

As expected, there is a critical trench width, x_{cr} , beyond which there is no effect of lateral boundaries on ultimate soil pressures, i.e. $p_{ult}/p_{ult,inf}=1.0$. It is noteworthy that the critical

width does not necessarily coincide with the maximum width of the failure mechanism, but it is generally somewhat larger (i.e. $x_{cr} \geq x_{max}$). This is attributed to the elastic deformations which develop on both sides of the failure surface and may also affect the development of soil pressures when constrained. In addition, it is observed that for lower than the critical trench width ($x < x_{cr}$), soil pressures increase substantially compared to the corresponding “infinite-trench” values, with the maximum difference being larger at shallow embedment depths (i.e. 6-8 times for $H/D=1.5$) than for deeper ones (i.e. 2-3 times for $H/D=11.5$). The effect of embedment depth may be attributed to the much larger width of the general shear (Type I) failure surface which develops at shallow depths as compared to the local shear (Type III) failure surface that will develop at large depths. Thus, the same shortening of the trench width will provide a more drastic constraint of the failure surface in the first case, leading to the observed much larger increase of soil pressures.

The numerical predictions in **Figure 5** may be analytically described by the following power relation:

$$\frac{P_{ult}}{P_{ult,inf}} = \left(\frac{x}{\alpha \cdot x_{max}} \right)^{-b_p} \geq 1.0 \quad (9)$$

The form of the above equation and the physical meaning of the associated parameters α and b_p are demonstrated in **Figure 6a**, for the random case of $H/D=6.5$ and medium dense backfill sand. Note that, when the ultimate pressure and the trench width axes are drawn in logarithmic scales, Eq. 9 implies a linear increase of normalized ultimate soil pressures with decreasing trench width ratio x/x_{max} , at a rate equal to b_p , while α corresponds to the critical value of x/x_{max} at which boundary effects become negligible. Based on the above interpretation, parameters α and b_p were subsequently back-calculated for each embedment depth, sand density and pipe diameter combination considered in the parametric investigation.

Figures 6b and **6c** show the correlation of parameters α and b_p with embedment depth ratio H/D . Different colors and symbols are used for loose and medium dense sand backfill, as well as, for pipe diameter $D=0.10$ and 0.70m . It is first observed that, in both figures, pipe size effects are efficiently accounted for by normalizing the embedment depth against the pipe diameter. It is further noted that α remains approximately equal to one for embedment depth ratios up to $H/D \approx 6.5$, implying that the critical trench width practically coincides with the maximum width of the failure surface at shallow and intermediate embedment depths. For the reasons discussed at the beginning of this section, α increases at larger depths to a final value which is approximately equal to $\alpha \approx 4.5$ for loose and $\alpha \approx 2.1$ for medium dense backfill sand. These trends may be analytically expressed as:

$$\alpha = \begin{cases} 2.7 + 1.8 \cdot \tanh[0.6 \cdot (H/D - 8.5)], & \text{for loose backfill sand} \\ 1.5 + 0.6 \cdot \tanh[0.6 \cdot (H/D - 8.5)], & \text{for medium backfill sand} \end{cases} \quad (10)$$

Exponent b_p is much less affected by the sand backfill density. Furthermore, it decreases with embedment depth ratio H/D , reminding that the effect of trench width is more severe for shallow pipeline embedment. In analytical form, the variation of b_p in **Figure 6c** may be described as:

$$b_p = 1.1 - 0.6 \cdot \tanh[0.32 \cdot (H/D - 3.2)] \quad (11)$$

Figure 7a provides an (one-to-one) comparison between analytical and numerical predictions of $p_{ult}/p_{ult,inf}$, while **Figure 7b** shows the associated relative error. Different symbols are used to indicate the three different modes of failure shown in **Figure 3**. The comparison shows a consistent agreement, regardless of failure mode, with less than 20% relative error for 93% of the data points.

3.2 Increase of ultimate displacements, y_{ult}

Using the same format as for ultimate soil pressures, **Figure 8a, b** and **c** show the variation of normalized displacements, $y_{ult}/y_{ult,inf}$ with normalized trench width, x/x_{max} . It is reminded that y_{ult} is defined (**Figure 4a**) as the lateral displacement at $p=p_{ult}$ when a bi-linear elastic-perfectly plastic relation is used to fit the p - y response of the soil springs. It is observed that, the qualitative trends identified for p_{ult} in the previous section, apply here as well. Namely, there is a considerable increase of ultimate displacements with decreasing trench width, while boundary effects may be effectively ignored beyond a critical trench width. In this case also, for a given trench width ratio x/x_{max} , the increase in ultimate pipe displacements is more pronounced for shallow embedment depth ratios H/D (**Figure 8a**) and can be ignored at large depths (**Figure 8c**).

Based on these similarities, the effect of trench width on ultimate displacements can be expressed in the same form as Eq. 9, i.e.:

$$\frac{y_{ult}}{y_{ult,inf}} = \left(\frac{x}{\alpha \cdot x_{max}} \right)^{-b_y} \geq 1.0 \quad (12)$$

Best-fit b_y values were obtained for the whole set of numerical data and subsequently related to the embedment depth ratio H/D (**Figure 9**). Different symbols are used for loose and medium backfill sand, as well as for $D=0.10$ and 0.70 m pipe diameters. It is again observed that scale effects are efficiently removed by normalizing embedment depth against the pipe diameter D . In addition, it is shown that b_y approaches zero for embedment depth ratios above $H/D \approx 8-10$, indicating that the effect of lateral trench boundaries becomes gradually insignificant at these depths. These trends are analytically expressed as:

$$b_y = \begin{cases} 0.55 - 0.55 \cdot \tanh[0.42 \cdot (H/D - 4.2)], & \text{for loose backfill sand} \\ 0.70 - 0.70 \cdot \tanh[0.35 \cdot (H/D - 5.5)], & \text{for medium backfill sand} \end{cases} \quad (13)$$

Figure 10a provides an (one-to-one) comparison between analytical and numerical predictions of $y_{ult}/y_{ult,inf}$, while **Figure 10b** shows the associated relative error. In this case also, the agreement between the two sets of predictions is a fairly consistent, with less than 20% relative error for 92% of the data points.

4. Effect of trench depth

To evaluate the effects of trench depth (distance d in **Figure 1**), a total of 200 analyses were performed for both loose and medium sand backfills, pipe diameters $D=0.10$ and 0.70m , as well as embedment depth ratios ranging from $H/D=1.5$ to 16 . In each case, the normalized trench depth varied from $d/D=0.15$ to a maximum value $d/D=3.0$ which corresponds to the “infinite-trench” response.

Interpretation of the numerical predictions revealed that the effect of trench depth is substantially less significant relative to that of the trench width. To show this, **Figure 11a** to **11c** summarize the computed variation with d/D of normalized ultimate soil pressures $p_{ult}/p_{ult,inf}$ and displacements $y_{ult}/y_{ult,inf}$, for three embedment depth ratios ($H/D=1.5, 6.5$ and 11.5), corresponding to Type I, II and III failure modes in **Figure 3**. The presentation format is the same as for the effect of trench width in **Figures 5** and **8**. Moreover, the same scale is retained in the vertical axes in order to highlight the large difference from the corresponding effects of trench width,

Observe that the effect of trench depth becomes substantial only for large H/D values (Type III failure), where both ultimate soil pressures and ultimate displacements increase by approximately 20-30%. In addition, any effects practically cease beyond a critical depth ratio $d/D\approx 1.0$. In view of the above observations, it was not considered necessary to develop detailed multi-variable analytical relationships for the effect of trench depth, and it is

alternatively proposed to use the constant correction factors for ultimate loads and displacements summarized in **Table 1**.

Table 1: Effect of trench depth on ultimate soil pressures and displacements

Backfill sand density	Embedment depth, H/D^*	$p_{ult}/p_{ult,inf}^{**}$	$y_{ult}/y_{ult,inf}^{**}$
Loose	<9.5	1.1 ± 0.1	1.0
	≥ 9.5	1.2 ± 0.2	1.2
Medium	<9.5	1.0 ± 0.1	0.8
	≥ 9.5	1.2 ± 0.2	1.2

**H: Embedment depth measured from the center of the pipe, D: Pipe diameter*

*** $p_{ult}/p_{ult,inf} = y_{ult}/y_{ult,inf} = 1.0$ for normalized depths $d/D \geq 1$ (d measured from the bottom of the pipe)*

5. Effect of trench wall inclination

The present section addresses potential boundary effects for the case where the walls of the trench are inclined, i.e. they are rotated by an angle $\theta < 90^\circ$ relative to the horizontal, as shown in **Figure 12**. Although not common in practice, this trapezoidal trench geometry is more consistent with the shape of the failure surface for shallow and intermediate embedment depth ratios in **Figures 3a** and **3b**, and may thus reduce the effects for vertical trench walls presented previously. Two scenarios were subsequently examined with regard to trench wall inclination, one for $\theta = \tan^{-1}(1/1) = 45^\circ$ and the other for $\theta = \tan^{-1}(2/1) = 63.4^\circ$. For each scenario both loose and medium sand backfill materials were considered, as well as embedment depth ratios $H/D = 1.5, 4.0, 6.5, 8.0$ and 11.5 . The normalized (semi-) width x varied from $x/D = 0.75$ to 16 , thus leading to a total of 120 additional numerical analyses. All analyses were performed for $D = 0.10\text{m}$, whereas the normalized trench depth below the pipeline was $d/D = 3.0$ so that so that the corresponding effects on ultimate pressures and displacements can be overlooked.

Figures 13a to **13c** illustrate the effect of trench inclination on the variation with trench width ratio x/x_{max} of the normalized ultimate soil pressures, $p_{ult}/p_{ult,inf}$, for $H/D = 1.5, 6.5$ and 11.5 . In each figure, the results for loose and medium backfill sand are shown separately,

while different symbols are used for the three different inclinations that were considered: $\theta=45^\circ$, 63.4° and 90° . As expected, the reduction of soil pressures with increasing side wall inclination is more pronounced for the shallow embedment depth ratio $H/D=1.5$, i.e. for Type I general failure mechanisms, where the $p_{ult}/p_{ult,inf}$ ratio can decrease by up to 2.5 times on average. This beneficial effect diminishes gradually with depth and becomes practically negligible for the large embedment depth ratio $H/D=11.5$, i.e. for Type III local failure mechanism. An additional important observation is that the largest part of the observed reduction is due to the inclination change from $\theta=90^\circ$ to $\theta=63.4^\circ$, while the additional benefit for further change to $\theta=45^\circ$ is marginal. This is explained by the fact that the inclination of the failure surface for Type I general failure mode ranges between 45° to 65° [eq. (7)], and consequently a reduction of the trench wall inclination to $\theta=63.4^\circ$ is sufficient to remove a major part of the lateral boundary constraint and the associated effects on ultimate soil pressures.

In view of the similar shape of $p_{ult}/p_{ult,inf}$ variation for the various trench wall inclinations, Eq. 9 may be modified to:

$$\frac{p_{ult}}{p_{ult,inf}} = \left(\frac{x}{\alpha \cdot x_{max}} \right)^{-b_{p,\theta}} \geq 1.0 \quad (14)$$

where the exponent $b_{p,\theta}$ is a function of the trench wall inclination

$$I_{\theta,p} = \frac{b_{p,\theta}}{b_p} \quad (15)$$

where exponent b_p corresponds to vertical trench wall ($\theta=90^\circ$).

The variation of correction factor $I_{\theta,p}$ with inclination angle θ is shown in **Figure 14** for various embedment depth ratios. Different symbols are used for loose and medium dense backfill sand. Consistent with the trends regarding the effects of trench wall inclination

discussed previously, $I_{\theta,p}$ approaches unity (no effect) as H/D increases, while the decrease of $I_{\theta,p}$ is much more significant when changing from the inclination angle from $\theta=90^\circ$ to 63.4° rather than from $\theta=63.4^\circ$ to 45° . Furthermore, the effect of backfill density is rather minor.

In analytical terms, correction factor $I_{\theta,p}$ can be expressed with the following analytical expression, drawn with black continuous line in **Figure 16**:

$$I_{\theta,p} = 1 - 0.35 \cdot \left\{ 1 - \tanh \left[0.32 \cdot (H/D - 6.3) \right] \right\} \cdot \sqrt{\cos \theta} \quad (16)$$

The effect of trench wall inclination on ultimate displacements is approximately the same with that on ultimate pressures. This is demonstrated in **Figure 15** which shows the variation of normalized initial stiffness of the load-displacement curve (i.e. $K=p_{ult}/y_{ult}$) for the inclined trench against the corresponding value for the vertical trench, i.e. $K_\theta/K_{\theta=90}$. The numerical data are presented separately for the three different failure modes, namely for backfill thickness ratios $H/D=1.5, 6.5$ and 11.5 , while different symbols are used for different sand densities and trench wall inclinations. It is observed that all data points in **Figure 15** are scattered around $K_u/K_{u,90} = 1.0$, implying that any change in ultimate pressures is directly reflected upon ultimate displacements. Hence, it may be readily assumed that, for trapezoidal wall sections with $\theta < 63.4^\circ$:

$$\frac{y_{ult}}{y_{ult,inf}} \approx \frac{p_{ult}}{p_{ult,inf}} \quad (17)$$

Eq. 17 implies that the effects of trench wall inclination on exponent b_y may be analytically expressed with the following correction factor:

$$I_{\theta,y} = \frac{b_{y,\theta}}{b_y} = 1 + (I_{\theta,p} - 1) \cdot \frac{b_p}{b_y} \quad (18)$$

where exponents b_p and b_y correspond to vertical trench wall ($\theta=90^\circ$).

6. Conclusions

A set of analytical relations is proposed for the computation of trench size and shape effects on backfill soil pressures applied during lateral displacement of pipelines embedded in stiff soils and rocks. Results from a large number (about 480) of elasto-plastic numerical analyses were used for this purpose, following calibration and verification against relevant small scale experiments. The detailed derivation of the analytical relations is described in previous chapters, while an application oriented summary is provided in the Appendix.

In conclusion, attention is drawn to the following main points:

- (a) The proposed relations provide essentially correction factors for the characteristics (p_{ult} , y_{ult}) of elastic-perfectly plastic Winkler soil springs for the analysis of pipelines embedded in trenches filled with sand. Hence, they can be combined with any rational method for the computation of these springs in the case of pipelines embedded in sand deposits with large ("infinite") lateral and vertical extend.
- (b) The numerical analyses were performed for trenches excavated within "rigid" natural geological formations. In practice, this condition is met only in the case of geological formations generally categorized as rocks, where the strength and stiffness is at least one order of magnitude larger than that of the sand backfill. For softer geological formations, the trench-induced increase in soil pressures will become more mild and consequently use of the proposed relations will lead to conservative pipeline design.
- (c) There is a minimum horizontal ($x_{cr}=a \cdot x_{max}$), between the displaced pipeline axis and the trench wall in the direction of lateral displacement, above which the trench size and shape will not affect the backfill pressures on the pipeline. In addition, there is no need for the proposed correction factors when computed pressures for the natural soil are less than those for "infinitely" extending back fill sand.

(d) Except from the cases described in (c) above, the correction factors of this study may help to optimize the trench size and shape so that pipeline strains due to permanent horizontal ground displacements (e.g from active fault rupture or landslides) are reduced to acceptable limits in a cost-efficient way.

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APPENDIX: Step-by-step computation of trench size and shape effects for pipeline design

The analytical relations developed in this paper are summarized below in a step-by-step application sequence. All symbols are explained in the main text.

- (a) Compute the ultimate soil pressure and displacement for natural ground conditions $(p_{ult}^{gr} \text{ and } y_{ult}^{gr})$ and for the backfill sand without trench effects $(p_{ult,inf}^{bf} \text{ and } y_{ult,inf}^{bf})$. In case that $p_{ult}^{gr} < p_{ult,inf}^{bf}$ skip the following steps, adopt the natural soil properties for the computation of the soil springs and proceed to the analysis of the lateral pipeline response.
- (b) Compute the minimum required horizontal (x_{cr}) and vertical (d_{cr}) distance of the displaced pipeline from the trench boundaries so that trench effects can be ignored:

$$d_{cr} = D \quad (b.1)$$

$$x_{cr} = \alpha x_{max} \quad (b.2)$$

where

$$\alpha = \begin{cases} 2.7 + 1.8 \cdot \tanh[0.6 \cdot (H/D - 8.5)], & \text{for loose backfill sand} \\ 1.5 + 0.6 \cdot \tanh[0.6 \cdot (H/D - 8.5)], & \text{for medium backfill sand} \end{cases} \quad (b.3)$$

and

$$x_{max} / H = 3.5 \cdot e^{-0.27 \cdot (H/D)} \quad (b.4)$$

or

$$x_{max} / D = \begin{cases} 3.0 + 0.10 \cdot (H/D)^{c_1} & \text{for } H/D > A \\ 13.1 - 1.2 \cdot (H/D)^{c_2} & \text{for } H/D > B \end{cases} \quad (b.5)$$

where c_1 , c_2 , A and B according to Table b.1

Table b.1: Estimation of constants c_1, c_2, A and B

Sand	$\gamma_{dry} (kN/m^3)$	c_1	c_2	A	B
Loose	14.8	1.9	1.1	6.0	10.0
Medium	16.4	2.40	1.70	4.8	9.5

- (c) For limited trench depth, i.e. when $d < d_{cr}$, compute correction factors $I_{d,p}$ and $I_{d,y}$ from Table c.1. Otherwise, use $I_{d,p} = I_{d,y} = 1.0$.

Table c.1: Estimation of correction factors $I_{d,p}$ and $I_{d,y}$

Sand	H/D	$I_{d,p}$	$I_{d,y}$
Loose	< 9.5	1.1 ± 0.1	1.0
	≥ 9.5	1.2 ± 0.2	1.2
Medium	< 9.5	1.0 ± 0.1	0.8
	≥ 9.5	1.2 ± 0.2	1.2

- (d) For limited trench width, i.e. when $x < x_{cr}$, compute correction factors $I_{w,p}$ and $I_{w,y}$ from the following analytical relations. Otherwise, use $I_{w,p} = I_{w,y} = 1.0$.

$$I_{w,p} = \left(\frac{x}{x_{cr}} \right)^{-I_{\theta,p} b_p} \geq 1.0 \quad (d.1)$$

where

$$b_p = 1.1 - 0.6 \cdot \tanh[0.32 \cdot (H/D - 3.2)] \quad (d.2)$$

and

$$I_{\theta,p} = 1 - 0.35 \cdot \left\{ 1 - \tanh[0.32 \cdot (H/D - 6.3)] \right\} \cdot \sqrt{\cos \theta} \quad (d.3)$$

$$I_{w,y} = \left(\frac{x}{x_{cr}} \right)^{-I_{\theta,y} b_y} \geq 1.0 \quad (d.4)$$

where

$$b_y = \begin{cases} 0.55 - 0.55 \cdot \tanh[0.42 \cdot (H/D - 4.2)], & \text{for loose backfill sand} \\ 0.70 - 0.70 \cdot \tanh[0.35 \cdot (H/D - 5.5)], & \text{for medium backfill sand} \end{cases} \quad (d.5)$$

and

$$I_{\theta,y} = \frac{b_{y,\theta}}{b_y} = 1 + (I_{\theta,p} - 1) \cdot \frac{b_p}{b_y} \quad (d.6)$$

- (e) Compute the ultimate soil pressure and displacement for the backfill sand including trench effects, as:

$$p_{ult}^{bf} = I_{dp} \cdot I_{wp} \cdot p_{ult,inf}^{bf} \quad (e.1)$$

and

$$y_{ult}^{bf} = I_{dy} \cdot I_{wy} \cdot y_{ult,inf}^{bf} \quad (e.2)$$

- (f) Finally, perform the pipeline analysis with the minimum ultimate pressure for natural soil (p_{ult}^{gr}) and backfill sand (p_{ult}^{bf}) and the associated ultimate displacement y_{ult} .

Figure Captions

- Figure 1:** Typical layout of the numerical model for "narrow" trench analysis: (a) before and (b) after application of lateral pipeline displacement $y=y_{max}$
- Figure 2:** Comparison of numerical results and experimental data for loose and medium dense sand and various embedment ratios
- Figure 3:** Failure modes during lateral pipeline displacement
- Figure 4:** Hyperbolic and a bilinear fitting on the numerical load-deformation (p-y) curve
- Figure 5:** Effect of trench width on ultimate soil pressures
- Figure 6:** Interpretation of numerical results for the development of an analytical expression for the effect of trench width on the ultimate backfill soil pressures
- Figure 7:** Evaluation of proposed relationships for the ultimate soil pressure: (a) One-to-one comparison between numerical and analytical data (b) Relative error
- Figure 8:** Effect of trench width on ultimate displacements
- Figure 9:** Interpretation of numerical results for the development of an analytical expression for the effect of trench width on the ultimate pipeline displacement
- Figure 10:** Evaluation of proposed relationships for the ultimate displacement: (a) One-to-one comparison between numerical and analytical data (b) Relative error
- Figure 11:** Effect of trench depth on ultimate soil pressures and ultimate displacements.
- Figure 12:** Configuration of the trapezoidal trench geometry
- Figure 13:** Effect of trench inclination on ultimate soil pressures for embedment depth (a) $H/D=1.5$, (b) $H/D=6.5$ and (c) $H/D=11.5$
- Figure 14:** Correction factor for exponent b_p in terms of trench wall inclination and embedment depth ratio
- Figure 15:** Effect of trench wall inclination on initial stiffness of load-displacement curve